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Irrigation canal system identification for control purposes

Carlos Sepúlveda Toepfer, José Rodellar Benedé

IOC-DT-P-2005-19
Setembre 2005

Institut d'Organització i Control
de Sistemes Industrials

Abstract

This report gives some guidance on how to obtain linear black-box models of irrigation canal reaches, using system identification techniques.

First of all, some general properties of the irrigation canal reaches are deducted, based on the use of the linearized Saint-Venant equations to model the water behavior.

Then different aspects of the system identification procedure like the sampling time, the model structure, the experiment design, etc., are studied, in order to avoid possible modelling problems and, in that manner, obtain a good linear model capable to be used in control systems designs.

The results obtained in the time domain and in the frequency domain show that one can achieve very accurate models, if the system identification procedure is designed with care having in mind the intrinsic properties of the system.

The research reveals that it is not convenient to perform a black-box irrigation canal system identification, without having a certain knowledge of the system.

Contents

Chapter 1

Introduction

Water irrigation canals are systems developed to transport water from main water storage reservoirs to several agricultural water-demanding farms, in irrigational seasons. Generally they cover very long distances; their range can go from hundreds of meters to hundred of kilometers, where farms are located normally close to them, but all the way along. To have a certain degree of command of the system, they normally have some control structures, like check gates, embedded in the water path, so as to regulate the amount of water flow in concordance with the water demands. With respect to the farmer's water offtakes, they are generally situated a few meters away from the upstream side of the gates, and the extractions are performed by pumps, weirs or any appropriate device.

As can be thought, it is not a trivial matter to manage this type of systems. They have to transport the water, minimizing the losses, and ensuring that each local farmer becomes their corresponding (exact when possible) amount of water at their corresponding frequency. Besides the inherent characteristics of the system don't help to much in reaching the objectives. The system presents very long time lags (from minutes to hours) in the transport of the water, delaying every decision applied on the system. Moreover, there are important water dynamical effects (generated, normally, by any change in the amount of water delivered), that produce, in different degrees depending on each case, interferences in the deliveries from the whole system (coupling). Historically, these problems and the availability of water have motivated the creation, in many countries, of irrigation associations with their own irrigation statutes and rules.

For these reasons many researchers have paid attention in improving the system's operational management by the automation of the system, applying the control engineering theory tools.

As commented in many papers about the matter (Clemmens et al., 1998; Malaterre et al., 1998; Sawadogo et al., 2000; Gómez et al., 2002; Rodellar et al., 2003), the goal of the automatic control of the operation is to maintain the water depth levels in the extraction zones, as constant as possible, by moving the intermediate check gates. This goal can be explained by the following reason: either if the irrigation water is taken out of the system by pumps or weirs, a constant level assures a constant supply of the water, eliminating flow variations to the farmers, and in that way, eliminating the coupling effect produced. In that way, several water demands can be fulfilled minimizing the interference between them.

1.1 About the importance of models in automatic control

The use of a good model of a process to be controlled is indispensable for almost all the currently existent control techniques (Shook et al., 1992). In addition to the possibility to test the control strategy to be implemented by means of computational simulation of the model, many control techniques use the model explicitly in the design stage of the controller or/and in the calculation of the control action (in this case check gates movements).

In the situation where the model is used in the design stage or in the computation of the control action, unfortunately, the complexity of the characteristics of the model (nonlinearities, delays, instability, etc), is, usually, directly proportional to the complexity of the control techniques and their implementation. For example, a linear and rational model opens the possibility to apply numerous and well-known linear control theories and standard techniques, which are relatively easy to implement. A nonlinear model, in contrast, requires much more effort and time to solve the control problem. Therefore it is always desirable to have the simplest model that can reproduce the behavior of the system. However, the ”closest to exact” behavior of a system is always obtained, when it can be obtained, by very complex models. For example all the existing processes are actually nonlinear. Linearity is only a simplification of the problem. So, models for control purposes should be made as a trade-off between simplicity and accuracy of the model, in concordance with the automation goal.

1.2 About irrigation canal models

Models that involve water are generally obtained making use of simplifications of the Navier-Stokes Equations, because of the complexity in dealing with them directly. For example for irrigation canals, one of the most accepted and used model in simulations, is the system given by the Saint-Venant Equations (Henderson, 1966), because of its capacity to represent the characteristics of real interest. However this system is a nonlinear partial differential equation system, which has analytical solution only in very special cases, having to utilize numerical methods to solve it properly. As model for computational simulation it is very accurate, but as model for control, it is clearly not appropriate for the reasons exposed before. That is the reason why, usually, linearizations or simplifications of the Saint-Venant equations are recurrently studied by the irrigation control research community (Schuurmans et al., 1995, 1999; Litrico and Fromion, 2004; Weyer, 2001). These models normally are physical models. That means that they are based on physical parameters of the real system. This has advantages and drawbacks. The advantages are that a physical model is reliable and has a very strong connection with the theoretical concepts. The main drawback is that it is necessary to feed the model with many parameters that have to be determined (from theoretical and/or experimental results) and an error in this determination can lead to a wrong modelling. For example, a bad determination or small variation of the Manning number (a coefficient related to the friction of the canals) can produce quite different results.

The scope of this work is to study the application of black box models to the problem of modelling irrigation canals for control purposes. These models do not necessary have a physical meaning; they only focus on reproducing the behavior of a system, and are obtained from input-output experimental collected data, in a process that is called System Identification.

Chapter 2

Model of a reach

2.1 Mathematical model

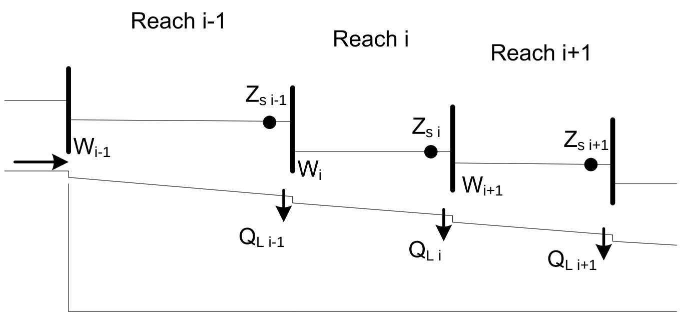


Figure 2.1: Irrigation canal schematic

A simplified vision of a typical irrigation canal can be observed in figure 2.1. As mentioned, it receives water form a source, and lets the water flow with a small slope. The intermediate check gates, represented by vertical lines, regulate with their openings (W_i) the desired flux, so as to maintain the water depth level ($Z_{s\,i}$) in the zones, where a water flow ($Q_{L\,i}$) is extracted for irrigational purposes.

For modelling intentions, a natural way of partitioning a canal is dividing it into reaches (also called pools). A reach is a portion of a canal between two check gates. So a normal canal can have several reaches with different characteristics (length, slope, width, etc.). However all the reaches share a common structure, focusing the problem of modelling an irrigation canal, in finding a suitable model for reaches. In that manner, the problem can be solved by a sum of the same model structure with only different parameter values.

In the specialized literature there are several approaches to obtain this model. In this work two facts will be taken into account: the location of the water extractions, generally near the reach's end, and the behavior's gradual change of the water when approaching to an obstacle like a cross-gate, resembling the characteristics of a water deposit. To put this practical knowledge in a mathematical model, it can be created an imaginary bound, which separates the reach, in an absolute manner, in a water transport area and in a water storage area (see figure 2.2). It should be noted that the location of the imaginary division is not so crucial; as it defines a particular size of the storage area, it will be correctly chosen whenever that area is remained small enough in relation to the reach's total size.

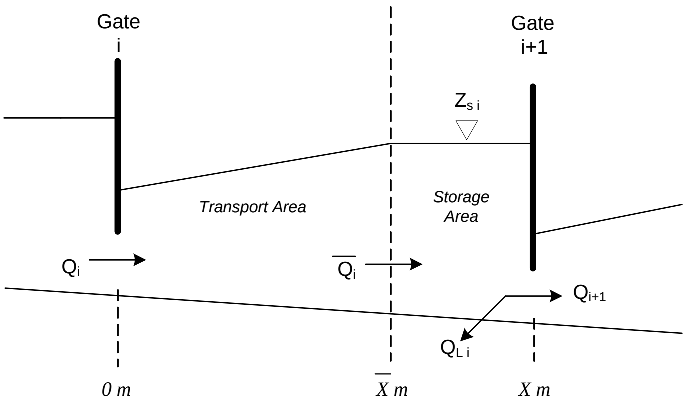


Figure 2.2: Simplified representation of a reach

Making use of the Saint-Venant equations (Henderson, 1966) to model the transport, and of the mass conservation principle to model the storage, that leads the following mathematical model:

- Transport

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{2.1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\frac{\partial Z}{\partial x} = gA(S_0 - S_f) \tag{2.2}$$

with initial conditions $Z(x,0) = Z_0(x)$, $Q(x,0) = Q_0(x)$, and boundary conditions $Z(0,t) = Z_i(t)$, $Q(0,t) = Q_i(t)$, $Z(\overline{X},t) = \overline{Z}_i(t)$, $Q(\overline{X},t) = \overline{Q}_i(t)$.

- Storage

$$\overline{Q}_i(t) - Q_{i+1}(t) - Q_{L\,i}(t) = \frac{dV_{s\,i}(t)}{dt} \tag{2.3}$$

In the transport equations (2.1)-(2.2), x is the longitudinal coordinate in the flow direction, t is the time, $A = A(x,t)$ is the wetted transversal section area of the canal, $Q = Q(x,t)$ is the volumetric water discharge, $Z = Z(x,t)$ is the water depth, S_0 is the bottom slope, $S_f = \frac{Q^2 n^2}{A^2 (A/P)^{\frac{4}{3}}}$ is the friction slope of the canal and g is the gravity acceleration. Besides, it should be noted that the wetted transversal section area (A) depends explicitly on the water depth (Z) and on the transversal section shape of the canal.

In the storage equation (2.3), $\overline{Q}_i(t)$ is the water flow that enters the area, $Q_{i+1}(t)$ is the water flow delivered to the next reach, $Q_{L\,i}(t)$ is the water flow extracted for irrigation purposes and $V_{s\,i}(t)$ is the water volume (that is a function of the water depth $Z_{s\,i}(t)$ and the geometry of that zone) stored behind gate $i+1$.

As can be seen from figure 2.2 and equations (2.1), (2.2) and (2.3), the variable values in the interface, denoted by an upper line $\left(\overline{Z}_i(t), \overline{Q}_i(t)\right)$, provide the link between both areas. Normally, they are not known a priori, but making use of extra mass and energy conservation relationships among that particular point and the described zones, the problem is solvable.

In summary, the model consist in a system of two nonlinear PDEs and one nonlinear (sometimes linear) ODE. Because of the reasons given before, the model is little advantageous for control purposes. In the search for a more convenient model, a model linearization around an operational condition should be performed.

2.2 Linearization of the model

Linearization is carried out replacing in the model, described by (2.1), (2.2) and (2.3), the expressions of the variables around a working point, namely, $Z(x,t) = Z_0 + z(x,t)$ and $Q(x,t) = Q_0 + q(x,t)$, and neglecting all the second-order terms (Litrico and Fromion, 2004). For the Saint-Venant equations (2.1) and (2.2), that yields:

$$\begin{aligned} B_0 \frac{\partial z}{\partial t} + \frac{\partial q}{\partial x} &= 0 &(2.4)\\ \frac{\partial q}{\partial t} + 2V_0 \frac{\partial q}{\partial x} - \beta_0\, q + \left({C_0}^2 - {V_0}^2\right) B_0 \frac{\partial z}{\partial x} - \gamma_0\, z &= 0 &(2.5)\end{aligned}$$

with $\gamma_0 = {V_0}^2 \frac{dB_0}{dx} + gB_0 \left[(1+\kappa) S_0 - \left(1 + \kappa - {F_0}^2 (\kappa - 2)\right) \frac{\partial Z_0}{\partial x}\right]$, $\beta_0 = -\frac{2g}{V_0}\left(S_0 - \frac{\partial Z_0}{\partial x}\right)$ and $\kappa = \frac{7}{3} - \frac{4S_0}{3B_0 P_0} \frac{\partial P_0}{\partial Z}$.

In (2.4) and (2.5), $F_0 = \frac{V_0}{C_0}$ is the Froude number, P_0 is the wetted perimeter, $C_0 = \sqrt{\frac{gA_0}{B_0}}$ is the water acceleration and $V_0 = \frac{Q_0}{A_0}$ is the water speed; all of them evaluated at the operation condition. The system, in this manner, has boundary conditions: $q(0,t) = q_i(t)$ $q(\overline{X},t) = \overline{q}_i(t)$, and $z(0,t) = z_i(t)$ $z(\overline{X},t) = \overline{z}_i(t)$.

One way of obtaining a solution for this system, is applying the Laplace transform and then reordering. That produces the following system of ordinary differential equations in the variable x, with a complex parameter s (the Laplace variable):

$$\frac{d}{dx}\begin{bmatrix} q(x,s) \\ z(x,s) \end{bmatrix} = A(x,s) \begin{bmatrix} q(x,s) \\ z(x,s) \end{bmatrix}^{1} \quad (2.6)$$

with $A(x,s) = \begin{bmatrix} 0 & -B_0(x)s \\ \dfrac{-s + \beta_0(x)}{B_0(x)\left(C_0(x)^2 - V_0(x)^2\right)} & \dfrac{2V_0(x)B_0(x)s + \gamma_0(x)}{B_0(x)\left(C_0(x)^2 - V_0(x)^2\right)} \end{bmatrix}$.

Because matrix A depends on the variable x, there is not a closed solution to the differential equation, and, therefore, it is necessary to use a numerical integration method to obtain the solution. Only the case where A is not dependent on x, has an analytical solution. This special case is called uniform regime, and is characterized by having the same water depth and the same water flow throughout a canal.

It has been proven in Litrico and Fromion (2004) that the problem can be solved numerically very efficiently, if it can be discretized by several "mini uniform regimes problems", in a way like in figure 2.3.

Using this approach, the solution to the linearized Saint-Venant equation system is given in the following manner:

$$\begin{bmatrix} q(\overline{X},s) \\ z(\overline{X},s) \end{bmatrix} = \Gamma\left(\overline{X},0\right) \begin{bmatrix} q(0,s) \\ z(0,s) \end{bmatrix} = \begin{bmatrix} \gamma_{11}(s) & \gamma_{12}(s) \\ \gamma_{21}(s) & \gamma_{22}(s) \end{bmatrix} \begin{bmatrix} q(0,s) \\ z(0,s) \end{bmatrix} \quad (2.7)$$

where the transfer function matrix Γ should be calculated using

[1] In the following any $f(s)$ will correspond to $\mathcal{L}\left\{f(t)\right\}$, the laplace transform of $f(t)$, which is a complex-valued function.

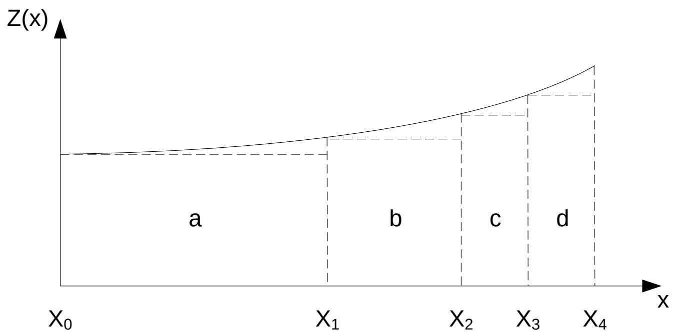


Figure 2.3: Schematic representation of an approximation by uniform regimes

$$\Gamma\left(x_n = \overline{X}, x_0 = 0\right) = \prod_{k=n-1}^{0} e^{A\,(x_k,\, s)\, h_k} \tag{2.8}$$

with $h_k = (x_{k+1} - x_k)$.

It should be noted that the term $e^{A(x_k,s)h_k}$ correspond to the exponentiation of a matrix (evaluated at x_k), which yields in this case:

$$e^{A\,(x_k,\, s)\, h_k} = \begin{bmatrix} \dfrac{\lambda_2(s)\, e^{\lambda_1(s)h_k} - \lambda_1(s)\, e^{\lambda_2(s)h_k}}{\lambda_2(s) - \lambda_1(s)} & \dfrac{c\left(e^{\lambda_1(s)h_k} - e^{\lambda_2(s)h_k}\right) s}{\lambda_2(s) - \lambda_1(s)} \\ \dfrac{\lambda_1(s)\, \lambda_2(s)\left(e^{\lambda_2(s)h_k} - e^{\lambda_1(s)h_k}\right)}{c\left(\lambda_2(s) - \lambda_1(s)\right) s} & \dfrac{\lambda_2(s)\, e^{\lambda_2(s)h_k} - \lambda_1(s)\, e^{\lambda_1(s)h_k}}{\lambda_2(s) - \lambda_1(s)} \end{bmatrix} \tag{2.9}$$

where

$$\lambda_{1,2}(s) = \frac{1}{f}\left(as + b \pm \sqrt{cs^2 + ds + e}\right)$$

with

$$\begin{aligned} a &= 2B_0(x_k)V_0(x_k) \\ b &= \gamma_0(x_k) \\ c &= 4{C_0}^2(x_k){B_0}^2(x_k) \\ d &= 4B_0(x_k)\left(V_0(x_k)\gamma_0(x_k) - \left({C_0}^2(x_k) - {V_0}^2(x_k)\right) B_0(x_k)\beta_0(x_k)\right) \\ e &= {\gamma_0}^2(x_k) \\ f &= 2B_0(x_k)\left({C_0}^2(x_k) - {V_0}^2(x_k)\right) \end{aligned}$$

In the solution expressed by (2.7) using (2.8) and (2.9), s is the laplace variable, $\overline{X}$ is the x position of the end of the transport area (beginning of the storage area) and Γ is the transfer function matrix that describes exactly all the input-output relationships of the linearized Saint-Venant equations in the laplace domain. In fact, it is possible with this formula to determine the value of the water flow and of the water depth at $\overline{X}$, by knowing their values at the beginning of the canal and by knowing the transfer function matrix.

Now, because the water depths generally are the outputs and usually the water flows are the inputs of an hydraulic model, it is convenient to express this matrix relationship in the same manner. This is performed with basic algebraic matrix manipulations, which in this particular case yields:

$$\left[\begin{array}{c} z(0,s) \\ z(\overline{X},s) \end{array}\right] = \left[\begin{array}{cc} -\dfrac{\gamma_{11}(s)}{\gamma_{12}(s)} & \dfrac{1}{\gamma_{12}(s)} \\ \gamma_{21}(s) - \dfrac{\gamma_{22}(s)\,\gamma_{11}(s)}{\gamma_{12}(s)} & \dfrac{\gamma_{22}(s)}{\gamma_{12}(s)} \end{array}\right] \left[\begin{array}{c} q(0,s) \\ q(\overline{X},s) \end{array}\right] \tag{2.10}$$

With (2.10), the water depth at the beginning and at the end of the transport area depend, in a deterministic manner, on the water flow that enters and that leaves the area.

Now, to complete the model it is necessary to linearize the storage equation given by (2.3). Following the same procedure as with the Saint-Venant equations, we obtain:

$$\overline{q}_i(t) - q_{i+1}(t) - q_{L\,i}(t) = \frac{dz_{s\,i}(t)}{dt} A_{s\,i} \tag{2.11}$$

Applying the laplace transform to (2.11) and replacing in it the expression for $z(\overline{X},s)$ from (2.10) assuming that $z_{s\,i}(s) \approx z(\overline{X},s)$, after reordering we obtain:

$$\boxed{z_{s\,i}(s) = \frac{\gamma_{21}(s)\,\gamma_{12}(s) - \gamma_{22}(s)\,\gamma_{11}(s)}{\gamma_{12}(s) - s\,\gamma_{22}(s)\,A_{s\,i}}\,q_i(s) - \frac{\gamma_{22}(s)}{\gamma_{12}(s) - s\,\gamma_{22}(s)\,A_{s\,i}}\,(q_{i+1}(s) + q_{L\,i}(s))} \tag{2.12}$$

(2.12) represents a linearized model (directly derived from the Saint-Venant equations) for a reach working around an operational point. It can be seen that the water depth of interest $z_{s\,i}$ (the one where the water is extracted for irrigation) can be obtained if the water flows that enter (q_i) and that exit the reach (q_{i+1} and $q_{L\,i}$) are known. However, in order to obtain the transfer functions that relates those variables, it is necessary the knowledge of a considerable amount of information, including the design parameters and the water depths of the reach, all of them at small enough (in order to reproduce correctly the desired characteristics of interest of the reach) longitudinal discrete positions of the reach.

Because the goal of this work is to design an appropriate and simple black-box (with no information about physical parameters) modelling procedure for a canal reach, that can fulfil the control automation requirements, this model is not going to be used explicitly. However, it would be used to study the main properties that a simpler model should have, and to decide which model structure is more adequate for the purposes established.

2.3 Properties of the linearized model

It is extremely important to know the transfer function characteristics of a system (e.g poles, zeros, etc.) for identification and control purposes. However only the uniform regime case has a clear analytical expression that can be analyzed. This is absolutely true, but an analytical expression can always be derived, using the fact that any shape of backwater curve of any type of reach can be well approximated using different number of terms in (2.8). In this manner, some properties of the general linearized model can be obtained, after developing the transfer functions terms of (2.12) for any number of discretization points (including the infinity). After some manipulations, it can be shown by mathematical induction that the structure of (2.12) will always be (no matter the reach or the operational condition) of the form:

$$z_{s\,i}(s) = \frac{1}{s}\left(\frac{n_1(s)}{d_1(s)\,d_2(s)}\,q_i(s) - \frac{n_2(s)}{d_2(s)}\,q_{i+1}(s) - \frac{n_2(s)}{d_2(s)}\,q_{L\,i}(s)\right) \tag{2.13}$$

In (2.13), $n_1(s)$ and $n_2(s)$ are irrational numerator expressions, $d_1(s)$ and $d_2(s)$ are irrational denominator expressions (all of them include exponentiation and roots of s polynomials) and $\frac{1}{s}$ correspond to an integration in the time-domain (integrator pole).

From the model structure (2.13), many things can be concluded. Some of them are the following:

- The existence of an integrator pole (real pole in the origin) denotes that the system is at best marginally stable. That means that the condition of stability, i.e. that any bounded input should produce a bounded output for all bounded initial conditions, does not necessary hold for this system. For system identification and control designs, this type of systems should be treated with special care; if not very bad performance behaviors could appear.
- The transfer functions related to the water flows that leave the reach, i.e. q_{i+1} and $q_{L\,i}$, are identical. That means that for model system identification, it is enough to identify the transfer function of one of them to know the other.
- The irrational terms of all the transfer functions imply that an approximation by rational transfer functions (with a Padé approximations for example) would be more or less accurate, depending on the number of terms used to approximate the irrationality. Hence it is expected to have much more terms in a rational approximated transfer function than in the original irrational one.
- The denominator $d_2(s)$ is contained in the transfer function that relates the water flow that enters the reach (q_i) with the water depth in the reach's end ($z_{s\,i}$). This means that several dynamical reactions are shared between all the input of the models. Technically speaking, some poles are shared by the transfer functions of the model.

All these properties could be a very useful guidance when searching for a simple modelization of a canal reach. It should be noted additionally that the produce of the integrator pole is not due the assumption of the storage zone. This pole also appears when not taking into account.

Chapter 3

System identification of the model

System Identification allows to build mathematical models of a dynamic system based on measured data, by adjusting parameters within a given model until its output matches the measured output as well as possible.

Although system identification techniques apply to very general models, the most common models are difference equations descriptions.

In the following sections, the main aspects of the system identification of a reach will be treated, in order to obtain an appropriate discrete-time model.

3.1 Discrete-time modelling issues

Continuous-time models and controllers are not directly implementable on digital computers, which require signals changes only at discrete time instants. For this reason, in most situations, it is a general practice to use discrete-time models or difference equations. However, the election of an adequate sampling time of the model and of a discrete system identification strategy is not trivial.

This section emphasizes some aspects that should be taken into account to obtain a good discrete-time model of a canal's reach.

3.1.1 Sampling time

The sampling frequency or sampling rate defines the number of samples per second taken from a continuous signal to make a discrete signal. The inverse of the sampling frequency is the sampling period or sampling time, which is the time between samples. Once decided, the model will only work with data collected exactly at that particular rate. In order to identify a black-box model from a data set, the time between samples should also be the same as the one of the intended model. Generally it is also recommended to sample faster than needed to filter the noisy components introduced by the measurement system, and then create a filtered data set at the chosen sampling frequency.

To choose the length of the sampling time, it is very important to review some of the theoretical and practical recommended rules related to the subject.

The Nyquist-Shannon sampling theorem states that the sampling frequency has to be, at least, twice the bandwidth of the signal being sampled. That means that the speed of the sampling should be at least twice the speed of the fastest dynamic of the system to be sampled. The rule of thumb is to choose the sampling frequency between 6 and 25 times the bandwidth frequency.

On the other hand, from the system identification research area, it is known that very fast sampling leads to numerical problems, model fits in high frequency bands, and poor returns for extra work (for example models with a huge amount of parameters) (Ljung, 1999). As the sampling interval increases over the natural time constants of the system, the variance of the estimated model parameters increases drastically. In view of system identification purposes, optimal choices of the sampling period for a fixed number of samples will lie in the range of the time constants of the system. These are, however, not exactly known, and overestimating them may lead to very bad results. All these aspects recommend a sampling frequency that is about ten times the bandwidth of the system. In practice it is useful to first record a step response from a system, and then select the sampling interval so that it gives 4-6 samples during the ”rise time” (time required to go from 10 to 90 percent of the final value).

Finally, if the model should be used for control purposes, several other aspects are significant. Generally the sampling interval for which the model is build should be the same as for the control application (to avoid the recalculation from one sampling interval to another). Besides a fast sampled model will often be non-minimum phase (there are unstable zeros in the transfer function), and a system with dead time may be modelled with delay of many sampling periods. Such effects may cause problems for some control design techniques. On the other hand, depending on the particular process and the control strategy used, it is necessary to have more information on a certain band of frequencies than in others, having to choose the sampling frequency so as to maximize the accuracy in that region. For example many frequency domain control techniques need an accurate representation of the frequency response at the gain crossover frequency, because of its relation with the maximum possible ”acceleration” of the closed loop system (plant+controller in feedback) before reaching the instability. Another design recommendation is to focus on the cutoff frequency ($-3\,\mathrm{dB}$ of input attenuation) of the desired bandwidth of the closed loop system.

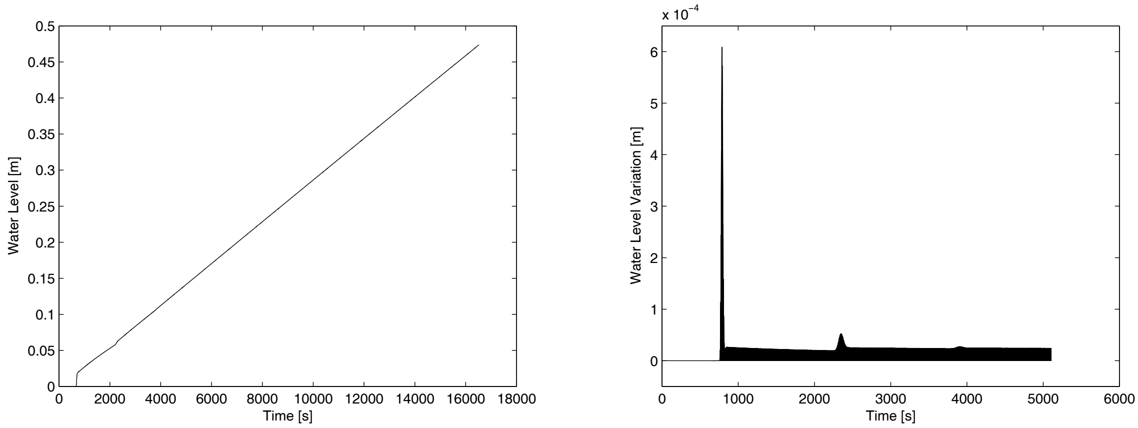
As can be seen, in order to choose a correct sample time for a control model, it is necessary to know, a priori, the main dynamical characteristics of the system in question and then apply any appropriate rule, with more or less care depending on each case.

To have a little insight of the difficulties that arise for a particular reach, in figures 3.1 and 3.2 it is shown the step response and the bode diagram of the theoretical transfer function that relates the water flow q_i with the water depth $z_{s\,i}$ in (2.12) for a pool with characteristics given by table 3.1.

Pool length	3000 m
Bottom slope	0.002 m/m
Bottom width	7 m
Pool shape	trapezoidal
Side slope	1.5 m/m
Manning's n	0.014
Operational flow	10 m^3/s

Table 3.1: Reach's Characteristics

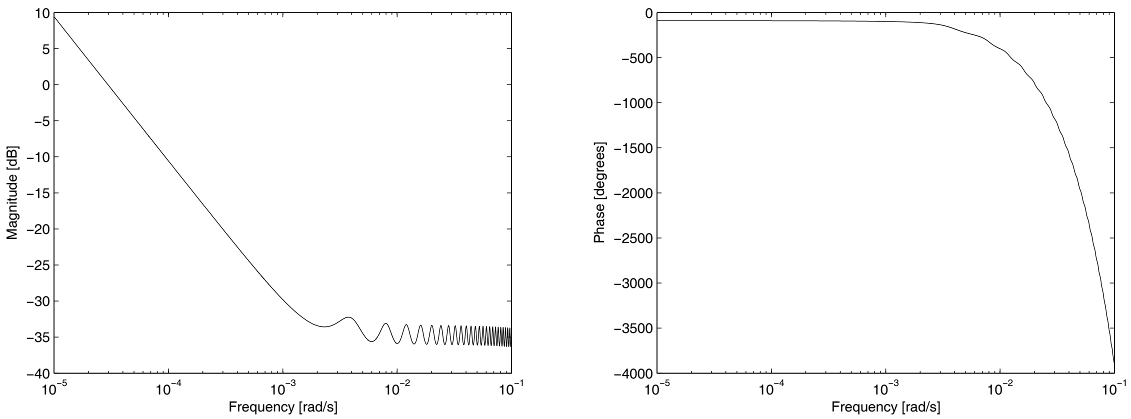
The step response is obtained simulating, in a numerically solved Saint-Venant model, the effect of a sudden increment in the water flow that enters a reach, maintaining the water discharges that exit the reach constant. The bode diagram is a logarithmic magnitude and phase plot of a transfer function, that gives information of the function evaluated in the s-plane imaginary axis, or in a more practical view, that shows what happens with the amplitude and the phase of the response of a system, when it is excited with a sinusoidal input at a given frequency. In this case is obtained numerically around the given operational backwater curve (by means of the linearized Saint-Venant equations), calculating the whole matrix product series (2.8) for each frequency point, and replacing their results in (2.12).



(a) Target Water Depth Response

(b) Derivative of the Water Depth Response

Figure 3.1: Water depth response for an Inlet flow step change for the reach of Table 3.1



(a) Bode Magnitude

(b) Bode Phase

Figure 3.2: Bode diagram between q_i and $z_{s\,i}$ for the reach of Table 3.1

The analysis of the step response and of the bode diagram for this particular configuration is the following.

Figure 3.1 shows, first of all, that after a sudden change in the flow that enters the reach, there is a delay in the reaction produced over the water depth of approximately 680 seconds. At that time a step increment occurs in the water depth, followed by a variable slope increment (including a quick mayor slope change at time 2300 seconds) that changes approximately until time 4000 seconds. At that time the water depth continue infinitely raising at a constant rate.

Looking in detail the step response obtained, one can conclude that there are small water depth variations (for example the first step water depth increment at 680 seconds) that, in order to be exactly reproduced, require the use of a very small sampling period (in order to have at least 4 points of that dynamical change, as recommended before). However, because the variations don't have a clear and predictable tendency until time 4000 seconds, a linear discrete model with a so short sampling period would have a great amount of parameters to include that knowledge, in order to be able to reproduce that type of behavior. So a trade off should be made in the election of the sampling period, between accuracy and model complexity, having in mind all the possible control design and numerical problems discussed before.

The bode diagram of figure 3.2 reveals that, around a working point, the linearized model has infinite number of poles (there are dynamics at infinite frequencies), and that the dominant pole is the pole at the origin (integrator pole) as derived before. Moreover the reach is oscillating with resonant modes.

The existence of infinite poles shows that it is impossible to reproduce the exact dynamical behavior of the system by sampling it, because of the fact that the sampling frequency should lie 10 times away from the highest frequency present in the system, in this case, the infinite. Moreover above the first resonant frequency, a linearized model with a higher sampling frequency would need more parameters than a model with a lower sampling frequency, to approximate the reach's behavior.

One possible choice for this particular case, selected having in mind the time prediction of the tendency slopes, can be a sampling time close to 221 seconds, obtained by the division of the stabilization time by 15 $\left(\frac{4000\,\mathrm{s}-680\,\mathrm{s}}{15}=221\,\mathrm{s}\right)$. With this sampling time a correctly obtained linear model would be able to reproduce the general tendency of the response properly. Moreover it would be able to reproduce properly at least the first resonant modes and the gain crossover frequency, and would have a delay of not so many sampling times (in this case 3), to facilitate the control design and avoid possible performance problems. However, it is probable that the model would have unstable zeros making the model to have a non-minimum phase behavior, because the sampling time does not divide the delay period exactly.

So, as can be seen, the election of the sampling period to identify a linear model and to reproduce properly the hydraulic behavior of a reach, should always be taken thinking in the particular system dynamics and in the intended purpose, using all the design knowledge at hand.

3.1.2 Discrete transfer functions

It is important to note, that a discrete-time model will never be the same as the continuous one; it will be only an approximation to a model with similar characteristics, despite the technique used.

In general, a discrete-time representation can be viewed as the z-transform of the discrete-time impulse response of a system for a given sample time, which in this case gives:

$$z_{s\,i}(z) = F_1(z)\,q_i(z) - F_2(z)\,q_{i+1}(z) - F_2(z)\,q_{L\,i}(z)^1 \tag{3.1}$$

[1] In the following any f(z) will correspond to $\mathcal{Z}\left\{f(kT)\right\}$, the laplace transform of a sampled time function $f(kT)$ (called z-transform), with $k=0,1,2,\ldots$ the sampling instant and T the sampling period. Besides for notation simplicity $f(kT)$ will normally be written as $f(k)$ only.

where z is the z-transform complex variable. For this reason it maintains a close relationship with the continuous Laplace transfer function obtained in (2.13). In that transfer function model, although it is general for any type of reach, there are two things that will always have a direct correspondence in the discrete-time representation:

1. Any Laplace domain pole will have a direct counterpart in the z-transform domain with the relation given by $p_z = e^{p_s T}$, where p_s is the pole in the Laplace domain, T is the sampling period and p_z is the discrete-time pole. Because of that, the pole at $s = 0$ (integrator pole) of $\frac{1}{s}$ will produce always a discrete-time pole at $z = 1$ (discrete-time integrator pole); that means that a $\frac{1}{z-1}$ will always appear in the z-transform of the model.

2. The time delays between the inputs (water flows) and the output (water depth of interest) of the continuous time model will satisfy the following property:

$$\mathcal{Z}\left\{f(k-d)\right\} = z^{-d}F(z)$$

where $f(k-d)$ is the delayed discrete-time impulse response of the model, k is the discrete time instant variable, d is the delay expressed in amount of instants and z is the z-transform variable. This implies that the z-transform model (3.1) will always appear in the following manner:

$$z_{s\,i}(z) = z^{-d_1}F_1'(z)\,q_i(z) - z^{-d_2}F_2'(z)\,q_{i+1}(z) - z^{-d_2}F_2'(z)\,q_{L\,i}(z) \tag{3.2}$$

where d_1 and d_2 are the time periods that takes each water flow to influence the water depth at the extraction zone, measured in discrete time instants.

Point 1 presents a property that have to be taken with care in order to avoid possible modelling problems. Dynamically and numerically it is very difficult to identify a discrete-time model with a pole exactly located at $z = 1$. The problem is that a small variation in the position will lead to a completely different dynamical model's behavior. With a pole at $z = 1.01$ the model would be unstable, making the model output go to infinite values after a small time period; with a pole at $z = 0.99$ the model would be strictly stable, producing that, with a constant input, the model output always reaches a constant value.

There are three approaches normally taken with respect to this problem when estimating a model by means of system identification:

- To forget about the problem and obtain a model anyway.
- To identify a model and afterwards correct the position of the pole in the estimated model.
- To acknowledge the existence of the pole and apply its influence directly to the data, in order to identify the other components of the model. This can be achieved in the following way:

$$y(z) = F(z)u(z) = \left[\frac{1}{z-1}F'(z)\right]u(z) \Rightarrow \begin{cases} y(z) &= F'(z)\left[\dfrac{1}{z-1}u(z)\right] &= F'(z)u'(z) \\ y'(z) &= y(z)(z-1) &= F'(z)u(z) \end{cases} \tag{3.3}$$

The first option is generally more recommendable (especially in the presence of noise) and is equivalent to make a cumulative sum of the input data, and the second option correspond to a differentiation of the output data. However it is important to note, especially when a maximally informative input signal was especially designed to identify a model, that the first option modifies the frequency characteristic of the input signal.

From the three presented approaches, it is clear that the last two are the better ones. The third one, however, has the disadvantage when modelling reaches, that because the sample time must be very small in order to reproduce the fastest dynamics adequately, larger sample periods could induce long-term modelling bias errors. So, in this case, the second option of identify and then correct would be the best when it is possible to perform. This is not always easily possible for some discrete-time model structures and, moreover, there are some types of model structures that cannot deal with processes with an integrator. For those cases the only choice is the third approach.

3.2 Discrete-time model structures

Usually linear discrete-time models can be divided into three main different classes:

1. Discrete transfer function models based only on the input-output characteristics of a system by means of the z-transform.
2. Discrete state-space models that incorporate the information related to all internal dynamics of a system directly in the time domain.
3. Orthonormal basis models that make use of the special properties of some basis functions to approximate systems.

Depending on the class and particular structure chosen, there is a whole list of parameter estimation methods that, with a given informative data set, can estimate the parameter values of that model in order to approximate a true system as well as possible. Examples of types of parameter estimation methods are Subspace methods for estimating state-space models, Prediction Error methods and Output Error methods for estimating transfer function models, etc..

In this work, only two model structures for reach modelling are considered: the ARX (Autoregressive with Exogenous Input) model, a transfer function based model, and the Laguerre model, an orthonormal basis based model. Both models have a finite number of rational elements, although the process is governed by an irrational transfer function. An approximation like that can be carried out because generally an irrational term can be approximated by a linear combination of rational ones, like in a Padè approximation of a function. However, depending on the irrational term, a good approximation could require a great number of rational terms to achieve good results.

All the model parameters are going to be estimated by a least-squares based algorithm in order to obtain a discrete-time approximation model of a reach.

3.2.1 ARX model

To derive the ARX model, it is first necessary to introduce the forward shift operator q and the backward shift operator q^{-1} respectively by:

$$qf(k) = f(k+1), \quad q^{-1}f(k) = f(k-1)$$

Then assuming that $F_1'(z)$ and $F_2'(z)$ of (3.2) can be approximated by two rational quotients of polynomials in the following manner:

$$F_1'(z) = \frac{B1(z)}{A(z)}, \quad F_2'(z) = \frac{B2(z)}{A(z)}$$

with

$$\begin{array}{rcl} A(z) & = & 1 + a_1 z^{-1} + \ldots + a_{na} z^{-na} \\ B1(z) & = & b1_1 + b1_2 z^{-1} + \ldots + b1_{nb1} z^{-nb1+1} \\ B2(z) & = & b2_1 + b2_2 z^{-1} + \ldots + b2_{nb2} z^{-nb2+1} \end{array}$$

and replacing in (3.2), that yields after reordering:

$$A(z)\, z_{s\,i}(z) = z^{-d_1} B1(z)\, q_i(z) - z^{-d_2} B2(z)\, q_{i+1}(z) - z^{-d_2} B2(z)\, q_{L\,i}(z)$$

Finally applying the inverse z-transform to move the problem backwards to the time domain, the general model structure for this case is:

$$A(q)\, z_{s\,i}(k) = B1(q)\, q_i(k - d_1) - B2(q)\, q_{i+1}(k - d_2) - B2(q)\, q_{L\,i}(k - d_2) \tag{3.4}$$

with

$$\begin{array}{rcl} A(q) & = & 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na} \\ B1(q) & = & b1_1 + b1_2 q^{-1} + \ldots + b1_{nb1} q^{-nb1+1} \\ B2(q) & = & b2_1 + b2_2 q^{-1} + \ldots + b2_{nb2} q^{-nb2+1} \end{array}$$

As can be seen, the parameters of the ARX model (3.4) are the polynomial orders na, $nb1$ and $nb2$, the polynomial coefficients, and the a priori known time delays d_1 and d_2 expressed in sampling instants. Once the parameters are determined, this model can calculate the downstream water level $z_{s\,i}$ at instant k, by a weighted sum of past values of the water level $z_{s\,i}$ and of past values of the water discharges q_i, q_{i+1}, $q_{L\,i}$, all of them previously collected at a given sampling interval T.

In some sense it is a little bit restrictive that both transfer functions, $\frac{B1(z)}{A(z)}$ and $\frac{B2(z)}{A(z)}$, have the same denominator polynomial $A(z)$, specially in this case when by means of (2.13) it is clear that they really don't have the same denominator. However (2.13) shows, in the same manner, that both transfer functions share some poles, so the assumption is not totally wrong. The reason for using the same denominator polynomial $A(q)$ have its roots in simplifying the parameter estimation process, because in that manner it can be performed with a linear least squares approach.

This model is capable to model stable and unstable processes so, as mentioned before, there exist two options in relation to the integrator parameter estimation problem. One is to estimate the model parameters and then fix the integrator pole location. This can be done obtaining the roots of $A(q)$, then replacing the root of $A(q)$ that is near to 1 by exactly a 1 and afterwards forming a new $A(q)$ polynomial keeping the other roots locations. The second option is to estimate the model parameters using the cumulative sum of each input model variable (water flows) and then multiply the estimated $A(q)$ polynomial by $(1 - q^{-1})$.

3.2.2 Laguerre model

This model is based on the Laguerre functions, a complete orthonormal set of functions in $L_2(0, \infty)$, the space of square Lebesgue integrable functions in the $(0, \infty)$ interval (Zervos and Dumont, 1988). These functions are described in the time domain by:

$$l_i(t) = \sqrt{2p}\frac{e^{pt}}{(i-1)!}\frac{d^{i-1}}{dt^{i-1}}\left[t^{i-1}e^{-2pt}\right]$$

where i is the order of the function ($i \geq 1$) and p is a positive parameter. The laplace transform of the Laguerre functions produces rational functions in the s variable of the following form:

$$L_i(s) = \sqrt{2p}\frac{(s-p)^{i-1}}{(s+p)^i}$$

By using a linear combination of a truncated number of these functions, any impulse response (or its associated transfer function) that belongs to the intersection of $L_1(0,\infty) \cap L_2(0,\infty)$, can be approximated as follows:

$$f(t) = \sum_{i=1}^{N} c_i\, l_i(t) = \mathbf{c}^T \mathbf{l} \qquad F(s) = \sum_{i=1}^{N} c_i\, L_i(s) = \mathbf{c}^T\, \mathbf{L}$$

$$\begin{array}{rcl} \mathbf{c}^T & = & \left[\begin{array}{cccc} c_1 & c_2 & \cdots & c_N \end{array}\right] \\ \mathbf{l}^T & = & \left[\begin{array}{cccc} l_1(t) & l_2(t) & \cdots & l_N(t) \end{array}\right] \\ \mathbf{L}^T & = & \left[\begin{array}{cccc} L_1(s) & L_2(s) & \cdots & L_N(s) \end{array}\right] \end{array}$$

A discrete-time state space version of this model can be obtained by applying a continuous network compensation method to each transfer function (Zervos and Dumont, 1988). The result of this operation yields:

$$\begin{array}{rcl} \mathbf{l}(k+1) & = & \mathbf{A}\,\mathbf{l}(k) + \mathbf{B}\,u(k) \\ y(k) & = & \mathbf{c}^T\,\mathbf{l}(k) \end{array} \tag{3.5}$$

where $\mathbf{l}(k)$ is the state vector of order N, $u(k)$ is the system input and $y(k)$ is the system output. Moreover, if T is the discrete sampling time, $\mathbf{A}$ and $\mathbf{B}$ are defined as:

$$\begin{array}{rcl} \mathbf{A} & = & \begin{bmatrix} \tau_1 & 0 & \cdots & 0 \\ \dfrac{-\tau_1\tau_2 - \tau_3}{T} & \tau_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \dfrac{(-1)^{N-1}\tau_2^{N-2}(\tau_1\tau_2+\tau_3)}{T^{N-1}} & \cdots & \dfrac{-\tau_1\tau_2-\tau_3}{T} & \tau_1 \end{bmatrix} \\ \mathbf{B}^T & = & \begin{bmatrix} \tau_4 & \left(-\dfrac{\tau_2}{T}\right)\tau_4 & \cdots & \left(-\dfrac{\tau_2}{T}\right)^{N-1}\tau_4 \end{bmatrix} \end{array}$$

with $\tau_1 = e^{-pT}$, $\tau_2 = T + \frac{2}{p}(\tau_1 - 1)$, $\tau_3 = -T\tau_1 - \frac{2}{p}(\tau_1 - 1)$, $\tau_4 = \sqrt{2p}\frac{(1-\tau_1)}{p}$.

Making use of the Laguerre model structure for a reach, the particular model gives:

$$\begin{aligned} z_{s\,i}(k) \;=\;& \mathbf{c1}^T \left[\mathbf{A1}\,\mathbf{l1}(k-1) + \mathbf{B1}\, q_i(k-1) \right] \\ & - \mathbf{c2}^T \left[\mathbf{A2}\,\mathbf{l2}(k-1) + \mathbf{B2}\, q_{i+1}(k-1) \right] \\ & - \mathbf{c2}^T \left[\mathbf{A2}\,\mathbf{l2}(k-1) + \mathbf{B2}\, q_{L\,i}(k-1) \right] \end{aligned} \tag{3.6}$$

In this type of model, the model output is obtained with only the inputs and the Laguerre functions values (always known for a particular p) at time $k-1$.

The parameters of the Laguerre-based model (3.6) are the coefficients vectors $\mathbf{c1}$ and $\mathbf{c2}$, and the designer-chosen number of Laguerre functions N and Laguerre pole (p) value (to calculate $\mathbf{Ax}$ and $\mathbf{Bx}$), one for each transfer function to be approximated. It is not necessary an a priori knowledge on the process orders or on any delay, but for increasing complexity transfer functions, it is necessary a higher number of terms in order to approximate adequately the required behavior. However the dynamical responses of each input can be adjusted in a totally independent manner.

This model can not approximate systems that are not strictly stable, so in this case the integrator should be passed directly to the data before proceeding with the system identification procedure. That can be realized through (3.3). Afterwards the integrator must be included in the obtained process model.

3.3 Experiment design: Input signal

An input signal should excite a system and have a rich frequency content, in order to successfully identify a model that approximates a true real system. A rich frequency content means that the input signal contains sufficiently many distinct frequencies. In practise, it is suitable to decide upon an important and interesting frequency band to identify the system in question, and then select a signal with more or less flat spectrum over this band.

Such an input is provided be the Pseudo Random Binary Sequence (PRBS) signal (Bialasiewicz, 1995). The sequence is generated by a digital waveform generator, which produces a binary signal by switching randomly between two output levels (a, $-a$). It owes its name pseudo-random to the fact that it is characterized by a sequence length within which the pulse width varies randomly, while it is periodic over a larger time horizon.

The PRBSs are generated by means of shift registers with feedback (implemented in hardware or software). The period is defined by the maximum sequence length:

$$L = 2^N - 1 \tag{3.7}$$

where N is the number of stages of the shift register. An example of a portion of a PRBS signal is presented in figure 3.3.

Assuming that $u(k)$ is a random binary process with the current value of a or $-a$ and that the value of $u(k)$ can change every T_{prbs} seconds, namely T_{prbs} is the switching period, the corresponding spectral density of the signal is:

$$S_{uu} = \frac{a^2}{\pi}\frac{T_{prbs}}{2}\left(\frac{\sin\left(\omega T_{prbs}/2\right)}{\omega T_{prbs}/2}\right)^2 \tag{3.8}$$

The spectral density function (3.8), can be assumed to be approximately flat up to a frequency about $0.3 f_{prbs}\,\mathrm{rad/s}$. If f_{prbs} is sufficiently high (as compared to the bandwidth of a plant to be identified), then the random binary process has a spectrum corresponding to broad-band noise.

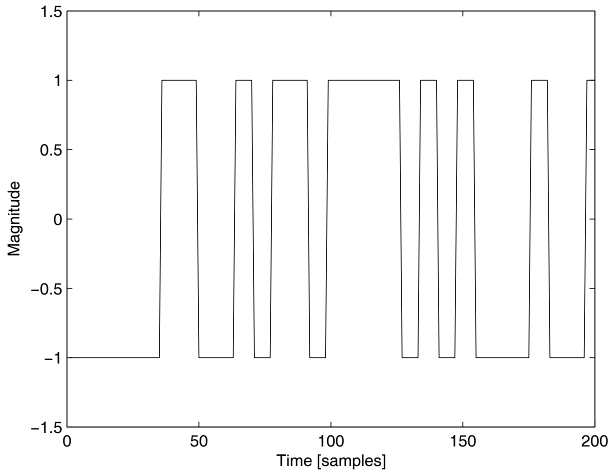


Figure 3.3: PRBS Test Signal

The spectrum of the pseudo-random binary signal is therefore an approximation for a broad band noise, provided that its clock frequency and its sequence length is large enough. However there are two thinks to satisfy in order to enjoy the good properties of this signal:

- One should work with an integer number of periods; that means: length{u(k)} $= nL$, with $n = \{1, 2, \ldots\}$. Usually one period of the signal meets the needs of the identification procedure.

- The amplitude of the signal a, must assure a good signal-to-noise ratio; normally ten times greater than the noise amplitude.

Additionally in order to correctly identify the steady-state gain of the process, the duration of at last one of the pulses in the PRBS must be greater than stabilization time of the process. As the maximum duration of a pulse is NT_{prbs}, N and T_{prbs} have to be chosen to adequately cover that time period.

As can be seen, if the PRBS sampling period T_{prbs} is chosen equal to a well-chosen process sampling period T_s (see section 3.1.1), the only way to augment the maximum duration of a pulse is increasing the number of registers N. However higher values of N lead to very large signal sequences, increasing the experiment duration and the data length; e.g. $N = 10$ produces a sequence length of 1023 data points, when $N = 12$ produces a sequence length of 4095 data points. In that case it is useful to choose the PRBS sampling period T_{prbs} to be a multiple of the process sampling period T_s: $T_{prbs} = pT_s$. The problem of this approach is that reduces the frequency range corresponding to a constant spectral density, so usually p is chosen to be $p \leq 4$.

From a system identification point of view, the data length ranges normally from 200 to 1000 data points, in order to have reliable values of the model parameters and less computational burden.

On the other hand, the proposed models are supposed to work only around an operational point of the reach, so to have locally rich data, all the movements induced to the system should be maintained sufficiently small. For normal systems this can be accomplished choosing a small enough PRBS amplitude a, but in this case that the system behaves like an integrator, it is additionally convenient that the experiment duration is kept as short as possible, to avoid that the system goes to far away from the working point.

3.4 Parametric identification

System identification can be defined as the process of obtaining a model for the behavior of a plant, based on the plant input and output data. If a particular model structure is assumed, the identification problem is reduced to obtaining the parameters of the model. The usual way of obtaining the parameters of the model is optimizing a function that measures how well the model, with a particular set of parameters, fits the existing input-output data. When process variables are perturbed by noise of a stochastic nature, the identification problem is usually interpreted as a parameter estimation problem. This problem has been extensively studied in literature for the case of processes which are linear on the parameters to be estimated and perturbed with a white noise (Ljung, 1999; Camacho and Bordons, 2004). That is, processes that can be described by:

$$z_k = \mathbf{\Theta}\,\mathbf{\Phi}_k + e_k \tag{3.9}$$

where $\mathbf{\Theta}$ is the vector of parameters to be estimated, $\mathbf{\Phi}_k$ is the vector of past input and output measures, z_k is the latest output measure and e_k is a white noise.

Once a model is written in a form like in (3.9), the parameters can be identified by using a least-squares identification algorithm.

All the models proposed in this work can easily be expressed as in (3.9) as follows:

3.4.1 ARX model

Assuming that the disturbances, that is, the differences between the measured output and the output calculated by the model, can be described by $e_i(k)$, a white noise zero mean sequence, model equation (3.4) can be rewritten as:

$$A(q)\,z_{s\,i}(k) = B1(q)\,q_i(k-d_1) - B2(q)\,[q_{i+1}(k-d_2) + q_{L\,i}(k-d_2)] + e_i(k) \tag{3.10}$$

Then solving for $z_{s\,i}(k)$, (3.10) yields:

$$z_{s\,i}(k) = A'(q)\,z_{s\,i}(k-1) + B1(q)\,q_i(k-d_1) - B2(q)\,[q_{i+1}(k-d_2) + q_{L\,i}(k-d_2)] + e_i(k) \tag{3.11}$$

with

$$A'(q) = (1 - A(q))\,q = -a_1 - a_2 q^{-1} + \ldots + a_{na} q^{-na-1}$$

This can be expressed as (3.9), by making

$$
\begin{aligned}
z_k &= z_{s\,i}(k) \\
\mathbf{\Theta} &= \begin{bmatrix} a_1 & a_2 & \cdots & a_{na} & b1_1 & b1_2 & \cdots & b1_{nb1} & b2_1 & b2_2 & \cdots & b2_{nb2} \end{bmatrix} \\
\mathbf{\Phi}_k &= \begin{bmatrix} -z_{s\,i}(k-1) \\ -z_{s\,i}(k-2) \\ \vdots \\ -z_{s\,i}(k-na) \\ q_i(k-d_1) \\ q_i(k-d_1-1) \\ \vdots \\ q_i(k-d_1-nb1+1) \\ -q_{i+1}(k-d_2) - q_{L\,i}(k-d_2) \\ -q_{i+1}(k-d_2-1) - q_{L\,i}(k-d_2-1)) \\ \vdots \\ -q_{i+1}(k-d_2-nb2+1) - q_{L\,i}(k-d_2-nb2+1) \end{bmatrix}
\end{aligned}
$$

3.4.2 Laguerre model

Assuming that the disturbances of the model can be described in the same manner as before, that is by $e_i(k)$, a white noise zero mean sequence, model equation (3.6) can be rewritten as:

$$
\begin{aligned}
z_{s\,i}(k) &= \mathbf{c1}^T \left[\mathbf{A1}\,\mathbf{l1}(k-1) + \mathbf{B1}\,q_i(k-1)\right] \\
&\quad - \mathbf{c2}^T \left[\mathbf{A2}\,\mathbf{l2}(k-1) + \mathbf{B2}\left(q_{i+1}(k-1) - q_{L\,i}(k-1)\right)\right] + e_i(k)
\end{aligned} \tag{3.12}
$$

Remembering that

$$
\mathbf{l}(k) = \mathbf{A}\,\mathbf{l}(k-1) + \mathbf{B}\,u(k-1) \tag{3.13}
$$

it is straightforward to show that (3.12) can be expressed as (3.9) in the following manner:

$$
\begin{aligned}
z_k &= z_{s\,i}(k) \\
\mathbf{\Theta} &= \begin{bmatrix} c1_1 & c1_2 & \cdots & c1_N & c2_1 & c2_2 & \cdots & c2_N \end{bmatrix} \\
\mathbf{\Phi}_k &= \begin{bmatrix} l1_1(k) \\ l1_2(k) \\ \vdots \\ l1_N(k) \\ -l2_1(k) \\ -l2_2(k) \\ \vdots \\ -l2_N(k) \end{bmatrix}
\end{aligned}
$$

As can be seen, in order to perform the parameter estimation procedure, first of all it is necessary to calculate the values of the N Laguerre functions responses at instant k for each model input, by means of the function (3.13). In that case the initial state vector should be $\mathbf{l}(0)^T = \left[\begin{array}{cccc} 0 & 0 & \cdots & 0 \end{array}\right]$.

Chapter 4

Results

To prove the effectiveness of the system identification of a reach, different models were identified for the reach with characteristics presented in table 3.1, by means of the parameter estimation of the proposed reach-model structures.

The initial water profile of the reach can be observed in figure 4.1.

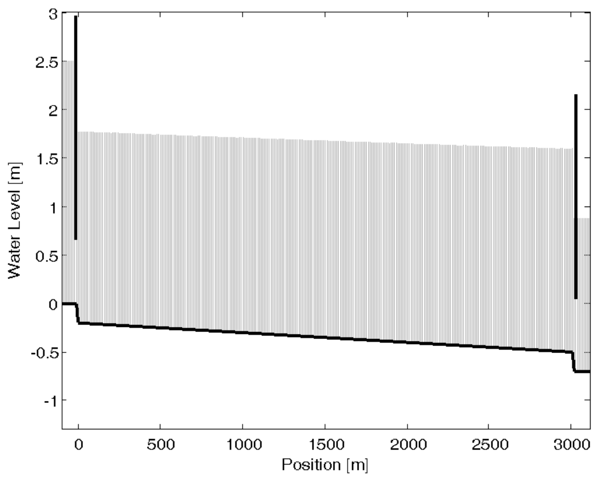


Figure 4.1: Initial water profile of the reach

Then following the recommendations made in section 3.1.1, the sampling period was chosen to be $215\,\mathrm{s}$, and an informative-rich input sequence was designed in order to excite all the relevant dynamics of the system. In this manner, two maximum length PRBS signals were designed in a form that the maximum width of the PRBSs was greater than the tendency-stabilization time of the step response of the reach: $4000\,\mathrm{s} - 680\,\mathrm{s} = 3320\,\mathrm{s}$. Besides having in mind "a not too large" sequence length, the following PRBS parameters were chosen: $p = 2$ and $N = 8$. Therefore the total sequence length was:

$$p \times L = 2 \times \left(2^8 - 1\right) = 510\,\mathrm{data\,points}$$

and the maximum pulse width was:

$$N \times T_s \times p = 8 \times 215\,\mathrm{s} \times 2 = 3440\,\mathrm{s} > 3320\,\mathrm{s}$$

In this manner, two PRBS input sequences that shifted randomly between $9.6\,\mathrm{m}^3/\mathrm{s}$ and $10.4\,\mathrm{m}^3/\mathrm{s}$ were generated by software, in order to apply them to the reach in the form of reach's input and output water discharges (q_i and $(q_{i+1} + q_{L\,i})$ respectively). The generated water discharges are depicted in figure 4.2 and figure 4.3.

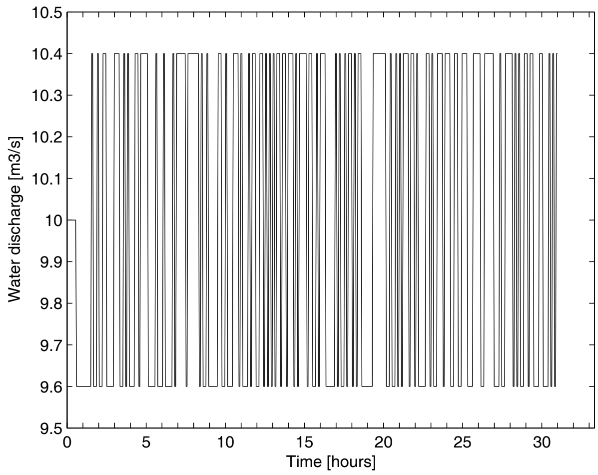


Figure 4.2: Imposed upstream water discharge

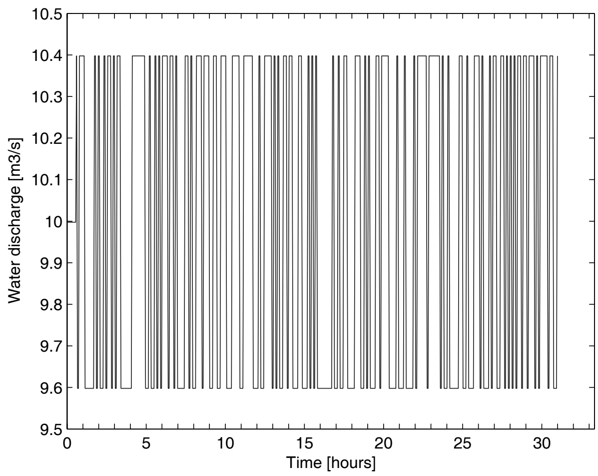


Figure 4.3: Imposed downstream water discharge

On the other hand, the reach's downstream water level ($z_{s\,i}$) response to the imposed water discharges can be viewed in figure 4.4.

As can be observed in figure 4.4, the experiment was designed in order to have the most information about

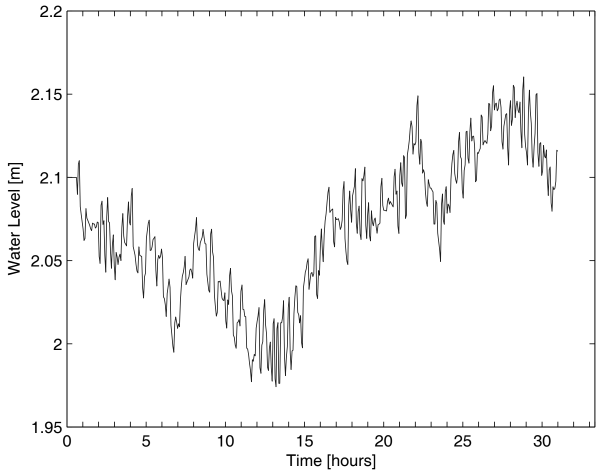


Figure 4.4: Reach's downstream water level

the system dynamics and the sufficient data points to feed the numerical parameter estimation algorithms, but maintaining the water level near the operational point ($2.1\,\mathrm{m}$), and so satisfying the linearity assumption.

With the generated data set, the following models were obtained:

- ARX model with integrator position correction:

$$\begin{aligned} A(q) &= 1 - 0.7791\,q^{-1} - 0.03589\,q^{-2} - 0.03576\,q^{-3} - 0.01423\,q^{-4} \\ &\quad - 0.01693\,q^{-5} - 0.01205\,q^{-6} - 0.2234\,q^{-7} + 0.1174\,q^{-8} \\ q^{-d_1}B1(q) &= q^{-4}\left(0.0229 - 0.0113\,q^{-1}\right) \\ q^{-d_2}B2(q) &= q^{-1}\left(-0.02598 + 0.01497\,q^{-1}\right) \end{aligned} \tag{4.1}$$

- ARX model with integrator after-addition:

$$\begin{aligned} A(q) &= 1 - 1.199\,q^{-1} + 0.1322\,q^{-2} + 0.01203\,q^{-3} + 0.03275\,q^{-4} \\ &\quad + 0.005332\,q^{-5} + 0.009966\,q^{-6} - 0.2101\,q^{-7} + 0.2167\,q^{-8} \\ q^{-d_1}B1(q) &= q^{-4}\left(0.02262 - 0.01995\,q^{-1}\right) \\ q^{-d_2}B2(q) &= q^{-1}\left(-0.02617 + 0.02631\,q^{-1} - 0.002822\,q^{-2}\right) \end{aligned} \tag{4.2}$$

- Laguerre model without integrator:

$$\begin{aligned} p1 &= 0.0115 \\ c1^T &= \begin{bmatrix} -0.0001 & -0.0017 & -0.0083 & -0.0168 & -0.0127 & -0.0032 \end{bmatrix} \\ p2 &= 0.0053 \\ c2^T &= \begin{bmatrix} -0.0015 & -0.0022 & -0.0014 & -0.0008 & -0.0007 & -0.0003 \end{bmatrix} \end{aligned} \tag{4.3}$$

In order to have an objective comparison, all model structures had 12 parameters to be identified from the data set information content. The orders of the polynomials and the delays of models (4.1) and (4.2) were obtained, comparing the results given by different combinations of them, and then picking the most appropriate ones. In the case of model (4.3), the Laguerre pole p was determined performing an optimal search of it, based on a Newton-Raphson iterative technique (Malti et al., 1998). All the parameters of the models where obtained using a least squares estimation algorithm. Model (4.1) includes the correction of the integrator pole position and model (4.2) was identified removing the integrator and then manually including it in $A(q)$. Model (4.3) is the Laguerre model obtained without integrator; the integrator had to be included augmenting the state-space model in an appropriate manner.

To compare the performances of the obtained models, they were tested in two different domains, the time and the frequency. A good performance in the time domain assures that the model can accurately model a behavior for a given input, while a good fit in the frequency domain assures that the identified model will approximate with good results, a response induced by a more general type of input.

In the time domain, the step response of a Saint-Venant modelled reach was compared against the step response of the obtained linear models. This was performed producing a step increase in the upstream water discharge, maintaining the downstream water discharge value constant. The results are given in figure 4.5.

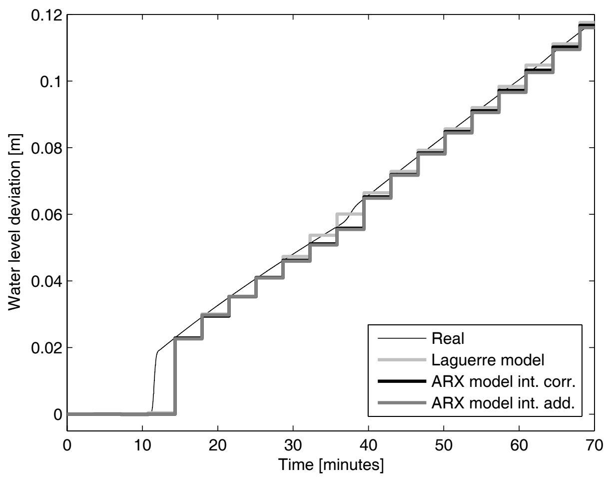
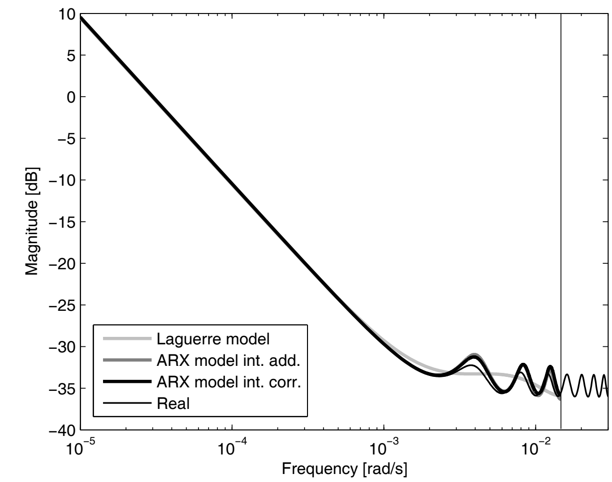


Figure 4.5: Step response of the reach v/s step response of the identified linear models

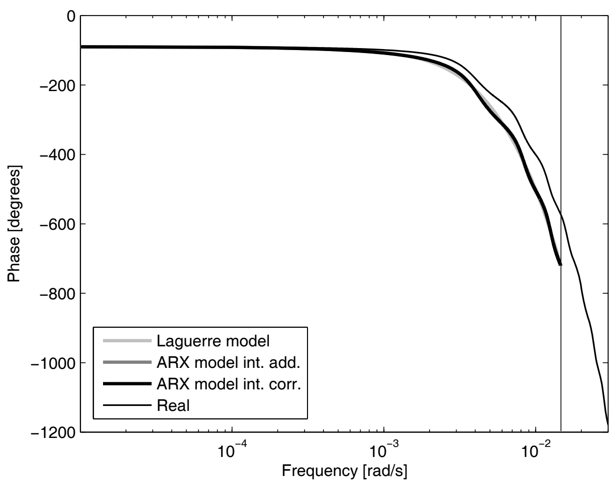
As can be observed in figure 4.5, all models performed relatively well in approximating the real behavior of the reach. Looking in detail, however, it can be seen that the Laguerre model was not so accurate in the transitions and that after a large time period, all models tend slowly to deviate from the real response.

To see if the models can perform only well for a step-type input or if they are really representing the system in question, the frequency response diagrams (or Bode diagrams) of the Saint-Venant modelled reach and of the obtained linear models are presented in figure 4.6 and figure 4.7, for the upstream flow discharge (q_i) - downstream water level ($z_{s\,i}$) relation and for the downstream flow discharge ($q_{i+1}+q_{L\,i}$) - downstream water level ($z_{s\,i}$) relation respectively.

Figures 4.6 and 4.7 show that all the models approximate very accurately the reach behavior at the low frequency region for both inputs, without having any magnitude or phase modelling error. In the high frequency region, the magnitude plots show that the ARX models can model with medium-high accuracy the position and magnitude of the resonant modes of the system until the Nyquist frequency (represented with a vertical line). In this area the Laguerre model tends to average the magnitudes of the frequencies, suggesting that the number of terms used are insufficient. In fact doubling the number of terms, the resonant modes approximation improves in a high degree, as can be seen in figure 4.8.

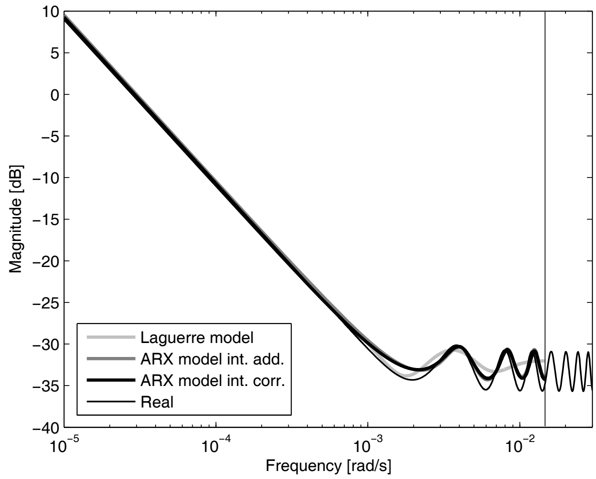


(a) Bode Magnitude

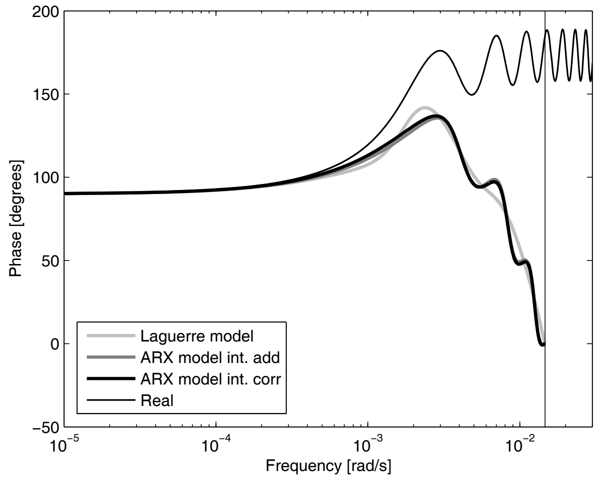


(b) Bode Phase

Figure 4.6: Bode diagram between q_i and $z_{s\,i}$ for reach of Table 3.1



(a) Bode Magnitude



(b) Bode Phase

Figure 4.7: Bode diagram between $(q_{i+1} + q_{L\,i})$ and $z_{s\,i}$ for reach of Table 3.1

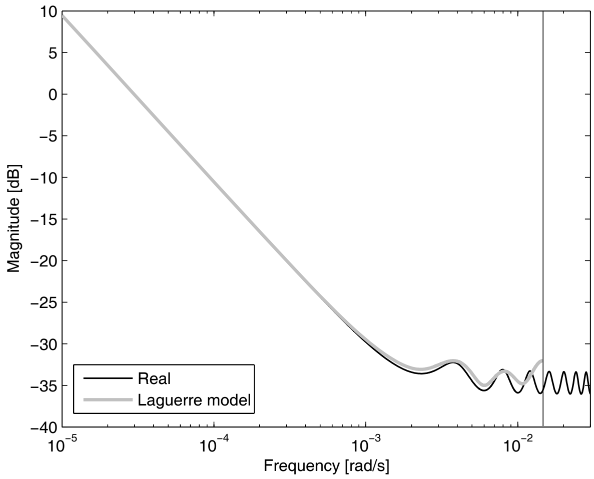


Figure 4.8: Bode diagram of Laguerre reach model with 24 parameters

On the other hand the phase plots at the high frequency region show that there are small phase errors, from 1/10 of the Nyquist frequency, that become more important when approaching to the Nyquist frequency. This is a normal behavior of some discrete-time models near the Nyquist frequency.

Chapter 5

Conclusions

Considering all the results presented before, the main conclusion of this work is that the linearized behavior of a canal reach can be approximated in good manner for control purposes by linear black-box models obtained by means of system identification techniques, in spite of the ”special” characteristics of the system in question, if some appropriate designing guides related to the system are followed. However, it is very important to remark that if the system identification procedure is performed in a blind manner, the obtained models could be very inaccurate and/or unappropriate and/or unstable. This is more true when the integrator presence is forgotten or when the sampling time is badly chosen.

Getting more in detail, there are some ideas crucially related to the problem in question. It has been proven that following some vastly extended modelling assumptions, the relationship between the downstream water level and the input and output water discharges has some common elements, that are independent of the reach configuration. These elements, observed under the frequency domain approach, are a particular basic structure, the irrationality of their components and an inherent integral behavior of the system.

Having in mind these particular characteristics, some of the system identification techniques were revised and adapted to the reach modelling problem.

The first problem to cope is the election of the sampling period. It is not a trivial matter and depends on a large degree on each particular case. It has been found that to perform a step response of the reach is useful to have an insight of the dynamical evolution of the water level, in order to, at least, keep track of the major tendency changes of the system. However this is only a point of view, and other interests, particulary those related to the control algorithm stability and complexity, can be more preponderant.

The identification experiment is one of the most important parts of the procedure, because it has to induce the process to show all its main dynamical characteristics in the generated data set. The information content has to be a priority, but without making the system go away to far away from the operating condition, because then the linearity assumption is not valid. In this case, because of the integrating property of a reach, the test should be maintained as short as possible, because of the probability of leaving the operating region. A pseudo random binary sequence (PRBS) is generally a good option, as demonstrated in this work. However, depending on the particular reach configuration or depending on the structural constrains of the water management devices, another type of signals could be more appropriate. Actually normal operation data of an irrigation reach can also be used, if it reaches a minimum threshold of information quality.

For all the models used in this work, the parameters can be easily obtained by linear least squares parameter estimation techniques. In fact they can easily be estimated online; that means that they can be estimated when the system is actually working, and, moreover, the algorithms can be programmed to perform adjustments in the models if the conditions of the system change, as well as in adaptive control strategies.

From the proposed models, the ARX models perform very well and better than the Laguerre model, despite of their inherent restrictions (linear model, same denominator polynomial, rational expressions, etc). Their structure (delays and orders) had to be chosen comparing the performance of different choices, but their simplicity makes the process very easy to accomplish. The results didn't show a real difference between the two ARX developed variants, but perhaps it is better to identify the model and afterwards correct the integrator position, because the process is not difficult to implement and then the data set is not filtered in any way.

The Laguerre model needed more parameters to cope with the ARX models, but it is a good alternative, specially because it doesn't require the knowledge of the delays or of the appropriate system orders; only the Laguerre poles have to be chosen. It directly approximates the system, and can give totally independent responses from each water flow discharges when required.

All the presented reach models can easily be used to generate a whole irrigation canal model. In that case, a multi-reach canal would be seen as a multiple input - multiple output (MIMO) system. So, the presented work is not only useful for decentralized reach controller designs: it can be still used when developing a centralized controller for an entire canal.

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